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FURTHER CONSIDERATIONS ON THE ORIGIN OF THE
 ZONE OF ASTEROIDS AND ON THE CAPTURE
 OF SATELLITES.

By T. J. J. SEE.

(Read November 4, 1910.)

In Volume II. of my "Researches on the Evolution of the Stellar Systems," 1910, which has just been published, I have treated at some length of the most important problems connected with the origin of the solar system, and have shown that as regards mode of origin the asteroids are connected with the periodic comets and have been gathered within Jupiter's orbit by the action of that great planet. This conclusion had been anticipated to some extent by the late Professor Stephen Alexander, of Princeton University, as far back as 1851, and more recently by the late Professor H. A. Newton, of Yale, and by the late M. Callandreau, of the Paris Observatory. The inferences of Newton and Callandreau resulted from their mathematical investigations of the perturbations of Jupiter upon small bodies crossing his orbit. Fortunately the weight of these eminent authorities is such that we need not dwell on the mathematical methods of reasoning employed. Our present aim is rather to examine briefly the consequences which follow from this theory, as developed in the second volume of my "Researches," and to make somewhat clearer the significance of certain observed phe-

nomena of the solar system, by an argument so brief and so much to the point that even a layman may grasp it without difficulty.

1. It was pointed out by Oppolzer in 1880¹ that the resistance to two homogeneous spheres revolving in a discontinuous medium of cosmical dust is inversely as their radii, and therefore relatively very large for a small body and very small for a large one.² The secular effect of such a cause, therefore, is to make the small body approach the sun very rapidly, while it scarcely modifies the mean distance of the large body, the latter change being so small that it often may be neglected entirely.

2. Now if we contemplate the arrangement of the orbits of the asteroids in the solar system, we find them grouped almost entirely within Jupiter's orbit, in accordance with the mathematical investigations of Newton and Callandreau, and moreover spread over the entire zone from Jupiter to Mars, and even extending beyond these limits. Thus Eros has a mean distance slightly less than that of Mars, while the Achilles group of asteroids projects beyond the orbit of Jupiter. How much wider the zone may hereafter be found to be, it is difficult to predict.

3. Since the asteroids have been thrown just within Jupiter's orbit by the successive actions of that great planet, and subsequently had their mean distances so decreased with the lapse of ages as to carry them down to the orbit of Mars, it follows that this spreading of the asteroids over such a wide zone affords a clear and unmistakable illustration of the effects of resistance and collisions—these small bodies having approached the sun much more rapidly than the giant planet Jupiter which gathered them in. No other interpretation can be given to the great width of the asteroid zone. For the perturbative action of Jupiter could throw the asteroids but slightly within his own orbit, and the great decrease in the mean distances of many of them must be accounted for in some other way. We conclude, therefore, that the more rapid dropping of the asteroids towards the sun illustrates the secular effects of resistance in the form of cosmical matter, such as meteoric swarms and comets,

¹ Cf. *A. N.*, 2314 and 2319.

² Cf. my "Researches," Vol. II., p. 293.

which must occasionally be encountered by the asteroids as well as by Jupiter. In the long run the secular effects of collisions with comets and similar *isolated bodies* is exactly the same as the effects of a medium of cosmical dust of nearly *continuous character*; so that we need not dwell on the character of the medium.

4. It is important to notice that as the small masses approach the center most rapidly, they would tend in time to overtake the larger planets nearer the sun. Thus the moon may have had originally a greater mean distance than the earth, but by degrees it was brought so near our planet that it passed under the earth's control and became a satellite. A similar conclusion holds for all the other satellites of the solar system. Besides crossing over the orbits of the larger planets, owing to larger eccentricity, these smaller bodies were originally at greater distances than their several planets, and in approaching them by degrees were at length brought within the range of the planetary attraction and captured, as explained in Volume II. of my "Researches on the Evolution of Stellar Systems."

5. In the PROCEEDINGS of this society, 1910, p. 213, we have explained the process of capture by a direct and simple method of reasoning, and in view of the considerations just adduced, one cannot doubt that this represents essentially the process of nature. The lesson taught by the great width of the zone of the asteroids is so very significant that it may serve as a practical demonstration of certain tendencies in the physical universe. The same conclusion may be otherwise verified, from a new and independent point of view, as follows.

It is shown by the exact data calculated from Babinet's criterion that the planets never could have been detached or thrown off from the central mass of our system, but were formed at great distances and have gradually neared the sun, as its mass increased and they revolved in the nebular resisting medium and gathered up more and more cosmical dust.

6. Since, therefore, the solar nebula as a whole did not rotate fast enough to detach the planets, or even exert a sensible centrifugal force, but they were originally independent nuclei formed in

the remoter parts of the nebula, at a great distance from the center, it will be doubly obvious that these insignificant secondary nuclei in the outer parts of the nebula could not have rotated rapidly enough to detach their satellites. This is emphasized also by the retrograde motions of the outer satellites of Jupiter and Saturn, which are entirely inconsistent with any theory of detachment. Such a view would be nothing less than absurd. The relatively greater energy of axial rotation of some of the planets is to be explained by the capture of nebulous matter circulating as a vortex in the powerful field of the sun's attraction, owing to its immense mass, which gives the particles impinging against the planet great relative moment of momentum.

7. The column of the accompanying table designated "centrifugal force" gives the fractional part of the centrifugal force due to rotation, when the central bodies are expanded to fill the orbits of their attendant bodies as imagined by Laplace. In this table the unit in each case is the amount of centrifugal force required to set the body revolving in its present orbit. The excessively small value of the centrifugal force due to the rotation of the expanded central mass is very remarkable; and emphasizes strongly the untenability of the view that the attendant bodies were thrown off. Nothing more misleading than this traditional view ever became current in the literature of science; and yet it still circulates in all old works on astronomy, and probably it will take many years to get it eliminated from the current thought of our times.

8. There is another impressive way of illustrating the untenability of the now abandoned detachment theory, as follows: It is shown in works on celestial mechanics that

$$v^2 = k^2(1 + m)(2/r - 1/a), \quad (1)$$

where k^2 is the constant of attraction, m the mass of the attendant body, that of the central mass being unity, r is the radius vector, a the semi-axis major, and v the velocity. This gives for an attendant body of insensible mass

$$a = \frac{1}{2 - \frac{v^2}{k^2}}. \quad (2)$$

As the orbits of the planets and satellites are essentially circular, we may put $r = a = 1$; and then since

$$a = \frac{1}{\frac{2}{r} - \frac{v^2}{k^2}} = 1,$$

it follows that in these units $v^2/k^2 = 1$. If instead of the velocity appropriate to carry the attendant body around in its nearly circular orbit, we substitute in (2) the fractional values of v^2/k^2 given in the above table, as due to the rotation of the central mass postulated by Laplace, we shall find that in all cases the semi-axis major of the new orbit is about one-half that of the existing orbit. The largest value results from the inner ring of Saturn, where the equation would give $a = 1/1.86 = 0.54$, the value of v^2/k^2 in this case being about 0.14.

TABLE OF DATA RELATING TO THE SOLAR SYSTEM.

Planet.	Centrifugal Force, calculated from data of Babinet's criterion, present orbital centrifugal force being unity.	Density of Central Body, when expanded to fill orbit, that of atmospheric air at sea level being unity.
Mercury.....	0.000000253	0.001776
Venus.....	0.000000134	0.0002723
The Earth.....	0.000000098	0.0001029
Mars.....	0.000000064	0.00002913
Ceres.....	0.000000035
Jupiter.....	0.000000019	0.000000732
Saturn.....	0.000000010	0.000000118
Uranus.....	0.0000000051	0.0000000146
Neptune.....	0.0000000032	0.0000000038

9. The obvious meaning of this result is that if the planets and satellites were projected along the tangents of their present orbits, with such small squared velocities as shown in the table, they not only would not pursue their present paths, but would in each case fall towards the centers and describe orbits with semi-axes just about one half. In other words, the projected satellites would pursue very elongated ellipses having their aphelia at the existing orbits, but their perihelia penetrating the central masses about which they now revolve. *In no case would over half a revolution be possible without collision, because the projected satellites would fall almost straight to the center and be absorbed in the central mass.*

If we compute the densities of the sun when expanded to fill the orbits of the planets, and of the several planets when expanded to fill the orbits of their respective satellites, we shall get the results given in the last column of the table. In considering this table it is sufficient to recall that an atmospheric pressure of 0.1 of a millimeter or a density of $1/7,600$ that of air is a very high vacuum;

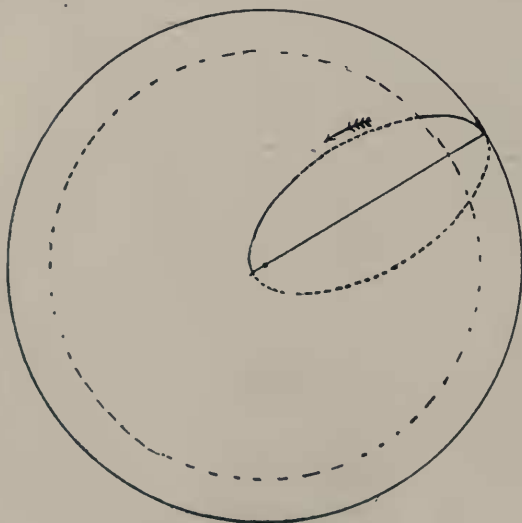


FIG. 1. Illustration of the motion of a particle projected along the inner edge of Saturn's dusky ring with velocity corresponding to planet's axial rotation when expanded to fill the ring. Pointed circle shows present dimensions of planet.

and since no greater hydrostatic pressure than this could be exerted from the center outward in case of most of the planets and many of the satellites as they fall unsupported by centrifugal force towards their dominant central masses, we see that these attendant bodies would in all cases fall practically without obstruction, and collision would in every case occur at the end of half a revolution. Could a more complete overthrow of the traditional detachment hypothesis of Laplace be imagined?

10. This affords an impressive illustration of the fallacy of the Laplacean theory of the origin of the planets and satellites; and

the only way we can explain the failure to detect this contradiction long ago is by the fact that Babinet's criterion seems to have been totally overlooked till taken up by the writer in 1908.

TABLE OF DATA RELATING TO THE SATELLITE SYSTEMS.

Planet.	Satellite.	Centrifugal Force, calculated from data of Babinet's criterion present orbital centrifugal force being unity.	Density of Central Body, when expanded to fill orbit, that of atmospheric air at sea level being unity.	
The Earth	The Moon	0.00005657	0.01965	
Mars.....	Phobos	0.001612	1151.	
	Deimos	0.000644	73.05	
Jupiter	V	0.034408	58.93	
	I	0.014694	4.66	
	II	0.009277	1.15	
	III	0.005820	0.285	
	IV	0.003323	0.0523	
	VI	0.0005416	0.000232	
	VII	0.0005217	0.000208	
	VIII	0.0002254	0.0000169	
Saturn	{ Inner Ring	0.1435 }	16.45	
	Mimas	0.048254 }		
	Enceladus	0.037651		7.61
	Tethys	0.030436		4.11
	Dione	0.023772		1.96
	Rhea	0.017017		0.717
	Titan	0.0073449		0.0576
	Hyperion	0.0060716		0.0324
	Iapetus	0.0025205		0.00232
	Phoebe	0.0006962		0.000049
Uranus.....	Ariel	0.0055888	2.40	
	Umbriel	0.0040111	0.88	
	Titania	0.0024454	0.200	
	Oberon	0.0018287	0.082	
Neptune.....	Satellite	0.0017177	0.43	

II. Since the attendant bodies could not be thrown off by rotation, and could not form where they now revolve out of mere scattered dust, owing to the feebleness of the mutual gravitation of such particles, under the powerful dispersive tidal action of the adjacent central masses, it follows incontestably that the planets and satellites have all been captured and added on from without. And we have explained the method of capture by simple and general considerations which make it certain that this process represents the true law of nature.

12. Any one who calculates the moment of momentum of orbital motion of a satellite such as the moon, which is 4.8 times larger than the present total moment of momentum of the earth's axial rotation, will perceive how powerfully the rotation of a planet may be accelerated by the impact of satellites against its surface. To be sure, the case of the moon is by far the most extreme in the solar system, owing to its large mass, but the tendency is the same everywhere, and really rapid rotations can be acquired only by the gathering in of quantities of satellites in the sun's field of force by large planets, as in the observed cases of Jupiter and Saturn.

The planets were formed at great distances from the sun, where the field of attraction would be much feebler than in their present situations; and hence nebulous matter collecting to such secondary centers in the outer parts of our nebula would give but feeble rotations of these masses about their axes; so that at no time in the past history of the solar system could a planet have rotated with sufficient rapidity to develop an appreciable tendency to detach a satellite. Such an hypothesis is wholly untenable, because it is found by calculation that the rapid rotations develop only when the planets are comparatively near the sun, and the relative moment of momentum of impinging particles therefore large.

Accordingly the simple considerations here adduced confirm from another independent point of view the results already obtained in second volume of my "Researches"; namely, that the moon and other satellites are merely captured planets, originally describing independent orbits about the sun; and show that the Capture Theory unquestionably is an ultimate law of nature.

U. S. NAVAL OBSERVATORY,
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